DOI 10.47176/sjis.5.3.1

Original Article

SRPH

SJIS, 2023; 5(3): 1-3

Comparison of some Iterative Methods for Finding Simple Root of Nonlinear Equations

Davood Nezami Behrooz

Department of Mathematics, Tuyserkar Branch, Islamic Azad University, Tuyserkan, Iran

***Corresponding Author:**

davood.nezami@gmail.com

Received: 11 July, 2023
Accepted: 28 August, 2023
Published: 15 September, 2023

ABSTRACT

We give a comparison of some iterative methods without memory for approximating a simple root of nonlinear equations presented in recent years. The efficacy of the present methods is tested on a number of numerical examples.

Keywords: Simple root, two-point iterative method, Kung and Traub conjecture, optimal order of convergence, computational efficacy.

Introduction

No doubts, with the advancement of digital computer, advanced computer arithmetic and symbolic computation, several scholars presented a good number of iterative methods without memory for approximating a simple root of nonlinear equations. A main tool for solving nonlinear problems is the approximation of simple roots x^* of a nonlinear equation $f(x^*) = 0$ with a scalar function $f: D \subset \mathbb{R} \to \mathbb{R}$ which is defined on an open interval D [1-4]. The Newton-Raphson method is one of the most widely used algorithms for finding roots. It is of second order and requires two evaluations for each iteration step, one evaluation of f and one of f'. Newton-Raphson iteration is an example of a one-point iteration, i.e. in each iteration step the evaluations are taken at one point. Multiple-point methods evaluate at several points in each iteration step and in principle allow for a higher convergence order with a lower number of function evaluations. Kung and Traub [5] conjectured that no multi-point method without memory with k evaluations could have a convergence order larger than 2^{k-1}. A multi-point method with convergence order 2^{k-1} is called optimal [6]. This paper is organized to present some two-point methods and convergence analysis of them with optimal convergence order four. Computational aspects, comparisons with other methods are illustrated.

Two Point Non-Optimal and Optimal Methods

In this section, we describe two-point methods of third and fourth order convergence for solving nonlinear smooth equations. Each member of the methods requires only three evaluations of the given function per iteration. Therefore, these methods with fourth-order convergence have efficacy index equal to 1.587 and agree with Kung and Traub conjecture. In 1960, the two-point

$$\begin{cases} y_n = x_n - \dfrac{f(x_n)}{f'(x_n)}, \\ x_{n+1} = y_n - \dfrac{f(y_n)}{f(x_n)-2f(y_n)} \dfrac{f(x_n)}{f'(x_n)}, \end{cases} \tag{1}$$

Ostrowski study [7] was the first multi-point of fourth-order. In terms of computational cost, each iteration of this method requires two evaluations of the function and one evaluation of its first derivative which is optimal according ti Kung and Traub's conjecture. As well as, the error equation of this method is

$$e_{n+1} = (c_2^3 - c_2 c_3) e_n^4 + \mathrm{O}(e_n^5).$$

King [3] developed an one-parameter family of fourth-order methods, which is written as

$$\begin{cases} y_n = x_n - \dfrac{f(x_n)}{f'(x_n)}, \\ x_{n+1} = y_n - \dfrac{f(x_n)+\beta f(y_n)}{f(x_n)+(\beta-2)f(y_n)} \dfrac{f(y_n)}{f'(x_n)}, \end{cases} \tag{2}$$

Publisher: Scientific Research Publishing House (SRPH), Shirvan, Iran
http://srpub.org
Email: sjis@srpub.org

where $\beta \in \mathbb{R}$ is a constant. The error equation of method (2) becomes

$$e_{n+1} = ((1+2\beta)c_2^3 - c_2c_3)e_n^4 + O(e_n^5).$$

In particular, the famous Ostroski's method defined in (1) is member of this family when $\beta = 0$.
Another one of two-point methods with optimal fourth order of convergence is Maheshwari method [6] to be seen below

$$\begin{cases} y_n = x_n - \frac{f(x_n)}{f'(x_n)}, \\ x_{n+1} = x_n + \frac{1}{f'(x_n)}\left(\frac{f^2(x_n)}{f(y_n)-f(x_n)} - \frac{f^2(y_n)}{f(x_n)}\right). \end{cases}$$
(3)

The error equation of method (3) which is given by

$$e_{n+1} = (4c_2^3 - c_2c_3)e_n^4 + O(e_n^5).$$

Kou et al. [4] presented a new method which is expressed as

$$\begin{cases} y_n = x_n - \frac{f(x_n)}{f'(x_n)}, \\ x_{n+1} = x_n - \theta\frac{f(x_n)+f(y_n)}{f'(x_n)} - (1-\theta)\frac{f^2(x_n)}{f'(x_n)(f(x_n)-f(y_n))}. \end{cases}$$
(2.4)

where $\theta \in \mathbb{R}$. The order of convergence of the methods defined by (4) is at least three, furthermore, if $\theta = -1$, then the order is four and error equation is $e_{n+1} = (3c_2^3 - c_2c_3)e_n^4 + O(e_n^5)$.

Chun method [1] is an example of fourth order multi-point methods which is given as:

$$\begin{cases} y_n = x_n - \frac{f(x_n)}{f'(x_n)}, \\ x_{n+1} = x_n + (1+\beta)\frac{f(x_n)+f(y_n)}{f'(x_n)} - 2\frac{f^2(x_n)}{f'(x_n)(f(x_n)-f(y_n))} - \beta\left(\frac{f(x_n)}{f'(x_n)} + \frac{f'(x_n)f(y_n)}{f^2(x_n)+f'^2(x_n)}\right). \end{cases} \quad (5)$$

with this error equation $e_{n+1} = (\beta c_2 + 3c_2^3 - c_2c_3)e_n^4 + O(e_n^5)$, which proves the fourth-order convergence for any real number β.

Numerical Performance

In this section we test and compare methods with a number of nonlinear equations. To obtain a high accuracy and avoid the loss of significant digits, we employed multi-precision arithmetic with 10000 significant decimal digits in the programming package of Mathematica [8-9]. In what follows, we are going to perform this kind of numerical experiments with the two test functions that appear in Table 1. In every case, and using the iterative methods described in the paper, we are going to reach the root x^*. starting in the point x_0. In Table 2, we compare two-point methods (1) − (5) together.

Table 1: Test functions f_1, f_2, f_3, root x^* and initial guess x_0.

Test function f_n	Root x^*	Initial guess x_0
$f_1(x) = \ln(1+x^2) + e^{x^2-3x}\sin(x)$	0	0.3
$f_2(x) = (1+x^3)\cos\left(\frac{\pi x}{2}\right) + \sqrt{1-x^2} - \frac{2(9\sqrt{2}+7\sqrt{3})}{27}$	$\frac{1}{3}$	0.4
$f_3(x) = \ln(x^2-x+1) - 4\sin(x-1)$	1	1.1

Table 2: Errors, COC and ACOC for test functions f_1, f_2, f_3 for methods (2.1) − (2.5).

$f_1(x)$ Methods	$\|x_1 - x^*\|$	$\|x_2 - x^*\|$	$\|x_3 - x^*\|$	COC	ACOC
(2.1)	0.741e−2	0.851e−8	0.139e−31	4.00000	4.00377
(2.2), ($\beta = 1$)	0.604e−2	0.173e−7	0.119e−29	4.00000	3.99806
(2.3)	0.514e−2	0.148e−7	0.104e−29	4.00000	3.99925
(2.4), ($\theta = -1$)	0.604e−2	0.173e−7	0.119e−29	4.00000	3.99806
(2.5), ($\beta = 0$)	0.338e−2	0.229e−8	0.482e−33	4.00000	4.00032

[DOI: 10.47176/sjis.5.3.1]

$f_2(x)$ Methods	$\|x_1 - x^*\|$	$\|x_2 - x^*\|$	$\|x_3 - x^*\|$	COC	ACOC
(2.1)	0.211e−4	0.345e−18	0.244e−73	4.00000	3.99999
(2.2), $(\beta = 1)$	0.712e−4	0.145e−15	0.254e−62	4.00000	3.99997
(2.3)	0.929e−4	0.566e−15	0.782e−60	4.00000	3.99998
(2.4), $(\theta = -1)$	0.712e−4	0.145e−15	0.245e−62	4.00000	3.99997
(2.5), $(\beta = 0)$	0.103e−3	0.935e−15	0.624e−56	4.00000	3.99996
$f_3(x)$ Methods	$\|x_1 - x^*\|$	$\|x_2 - x^*\|$	$\|x_3 - x^*\|$	COC	ACOC
(2.1)	0.830e−6	0.220e−26	0.108e−108	4.00000	3.99999
(2.2), $(\beta = 1)$	0.192e−5	0.190e−24	0.182e−100	4.00000	3.99997
(2.3)	0.661e−4	0.186e−16	0.118e−66	4.00000	3.99999
(2.4), $(\theta = -1)$	0.192e−5	0.190e−24	0.182e−100	4.00000	3.99999
(2.5), $(\beta = 0)$	0.391e−4	0.811e−18	0.150e−72	4.00000	3.99999

CONCLUSION

Some optimal two-point methods without memory and their numerical comparison have been presented in this review paper. These methods based on the Newton methods use only three function evaluations per iteration. Therefore, they are optimal.

REFERENCES

1. Chun, C., A family of composite fourth-order iterative methods for solving nonlinear equations, Appl. Math.Comput., 187, 951 - 956 (2007)
2. Cordero, A., Torregrosa, J.R., Variants of Newton's method using fifth-order quadrature formulas, Applied Mathematics and Computation, 190, 686 - 698 (2007)
3. King, R.F.: A family of fourth order methods for nonlinear equations, SIAM J. Numer. Anal. 10, 876 - 879 (1973)
4. Kou, J., Li, Y., Wang, X., A composite fourth-order iterative method, Appl. Math. Comput., 184, 471 - 475 (2007)
5. Kung, H.T., Traub, J.F., Optimal order of one-point and multipoint iteration, J. Assoc. Comput. Math., 21, 634 - 651, (1974)
6. Maheshwari, A.K., A fourth-order iterative method for solving nonlinear equations, Appl. Math. Comput., 211, 383 - 391 (2009)
7. Ostrowski, A.M.: Solution of Equations and Systems of Equations, Academic Pres, New York (1960)
8. Traub, J.F., Iterative Methods for the Solution of Equations, Prentice Hall, New York, (1964)
9. Weerakoon, S., Fernando, T.G.I., A variant of Newton's method with accelerated third-order convergence, Applied Mathematics Letters, 13, 87 - 93 (2000)

SJIS

Citation: Nezami Behrooz D. Comparison of some Iterative Methods for Finding Simple Root of Nonlinear Equations. SJIS, 2023; 5(3): 1-3.

https://doi.org/10.47176/sjis..5.3.1

[DOI: 10.47176/sjis.5.3.1]