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Exponential Stability and L_2 Gain Analysis for Systems with Infinite Distributed Delay by Scalar Kernels to Track a Surface Vessel by Submarine

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ABSTRACT

In this paper, the problem of trajectory optimization in tracking the location of surface moving targets by measuring the side angle alone is studied. The performance of target tracking with side angle alone depends on the stability of the target position in the target observer's motion path or the optimal observer maneuver. First, the modeling of the path control problem is performed by the kernel-scalar matrix method. Then, by analyzing the L_2 interest rate, the control law is obtained for moving independently of the initial conditions. The advantages of the proposed modeling are maximizing the delay limit for the stability of the entire maneuver time, calculating the control rule at the start of the maneuver and high flexibility in applying the travel restrictions. The efficiency of the method presented by simulation with scalar kernel matrix method with control methods of delayed systems with distributed delay is shown and compared by recent references. Performance is also evaluated in different scenarios and its reliability is checked. This method is also used in the practical problem of tracking a surface vessel by submarine.

Keywords: Exponential stability, L_2 interest rate analysis, Infinite distributed delay, Scalar kernel, Side tracking alone

Introduction

Target tracking In situations where there is a limit to measuring target parameters with active sensors, it is done only by measuring the angle of the target by inactive sensors, which is called BOT Bearings-Only Tracking. It has many special applications in military and commercial industries [1]. The BOT problem is categorized according to the number and position of the senses, the number of targets, and the motion dimensions of the targets. In many practical problems, the initial position and condition of the target is also unknown in the BOT problem [2]. Also, pursuit with the least maneuver due to the limitations and obstacles of the observer's movement path, is one of the basic requirements of this special practical issue in practice. High maneuverability produces acoustic noise and greater observer visibility [3]. In recent decades, the issue of BOT and TMA Target Motion Analysis has been a topic of interest for researchers [4]. This has been used for acoustic issues (submarines with passive sonar), electromagnetic equipment (ESM sensors), and optical equipment (for satellites and infrared cameras). Much research has been done on various types of BOT issues, which can be referred to the important books on this subject in references [5-7].

The standard version of side motion analysis (BOTMA) consists of two animations on a two-dimensional surface in which the observer (pursuer) and the target move at a quasi-linear velocity at a constant speed and direction during tracking time. Thus, the meaning of classical BOTMA is the calculation of four parameters, including the two coordinates of geographical position, speed and course of movement (target), which is done by collecting and measuring the target side by the observer of the observer [8]. Under this classical assumption, if the

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velocity vector is constant, the observer cannot identify the target and therefore the problem is not stable. In the last decade, the effect of maneuvering on increasing target stability in BOT has been discussed. When stability is definite, due to the combination of side measurement with error, BOT accuracy is highly dependent on maneuver [9]. Reference [10] shows that maximizing the delay limit of the BOT problem is achieved by the interaction of two reciprocal conditions, one is the reduction of the distance from the observer to the target and the other is the orthogonal motion of the observer on the target line of view. In some cases, researchers have used approximate s-track maneuvers to meet the stability requirement of these delayed distributed systems with extra-delayed delay, but this, in addition to generating acoustic noise, incurs additional path costs [11].
The main tasks of maneuver control are done to position the target (stationary target). In the reference [12], the movement of the carrier on the two straight paths of the line has been investigated and the effect of the movement path has been compared. Assuming a fixed course in the first step of the route, the delay rate course is calculated in the second step of the route in order to maximize the accuracy of calculating the target distance by increasing the L_2 gain. In the reference [13], for a positioning problem, by defining the Lyapunov function of the proximity constraint, the problem of standard kernel matrices for the BOT problem is presented. In [14], using change calculation methods (indirect methods), the closed-loop system for optimal maneuver (dependent on the input parameter of the coefficient of approximation) in the positioning problem (static target) has been studied. In recent references, the necessity of establishing these two reciprocal conditions has defined the path of the observer in applied problems as a spiral (Figure s). The importance of this method, in addition to the simplicity of online calculations, is that the calculations do not depend on the uncertainty of the initial conditions. The cost function of this method by L_2 gain is the long and undesirable path of the observer maneuver, the possibility of instability in some directions of movement such as the target, and the non-optimization of the motor end time relative to the maneuver mud [15]. The minimal trajectory of the observer in the direction of approaching the target, in addition to improving the condition of the infinitely distributed delay limit in the far-reaching scenarios of the target, causes an accurate estimate of the end time of the maneuver. As a comprehensive example of the indirect solution method, in the reference [16], by forming a Hamiltonian boundary value (HBVP) problem and using the theory of variance calculus, a relationship between the direction of the observer and the angle of the target in the optimal path is presented. Is. The difficulty of presenting the final boundary conditions and knowing the initial position is one of the limitations of this method. On the other hand, methods that have examined the timing of maneuvers recommend approaching the target at the beginning of the route and bypassing the target at the end of the route [17].
Compared to existing studies, this is the main contribution of this article. In the second part, the definitions and modeling of the BOT problem are stated. In the third step, the exponential stability of distributed delayed systems with infinite delay and L_2 gain analysis by standard kernel matrices in the form of Lyapunov function and LMI extraction is described. In the fourth section, in fact, the simulation of the article is presented, and finally, to show the correctness of the theoretical results and the effectiveness of the algorithms, in the fifth section and the final section of the article, the general results of the article are stated.

Definitions and Modeling of the BOT Problem

Figure 1 shows the two-dimensional geometry of the BOT problem [18]. The variables used in the article according to Figure 1 are:

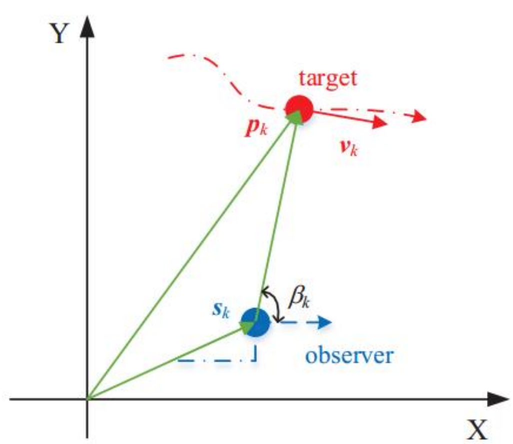

Figure 1. Two-dimensional geometry of the BOT problem [18]

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General Problem BOT Estimation of the path of the target (speed and position of the target) is defined by measuring the sensitive data on the maneuverable side while maneuvering (sensitive carrier). Next, the Cartesian coordinate system is used to mathematically define the BOT problem in the target mode with constant speed and direction (without maneuver). Therefore, objective modeling and BOT interception problem equations with nonlinear measurement function of BOT problem at t moment are defined as the following relations [19].
The target state vector is defined by the linear velocity V_t and the initial vector V_0 by equations (1) and (2).

$$x_k^t = [x_k^t, y_k^t, \dot{x}_k^t, \dot{y}_k^t]^T \quad (1)$$
$$x_k^0 = [x_k^0, y_k^0, \dot{x}_k^0, \dot{y}_k^0]^T \quad (2)$$

As a result, the relative motion vector of the target is obtained as (3).

$$x_k = x_k^t - x_k^0 = [x_k, y_k, \dot{x}_k, \dot{y}_k]^T \quad (3)$$

The dynamic equations of the target state space, at a constant speed, assuming the acceleration is unknown and applying it to the model as Gaussian independent noise (4) are as follows.

$$W_k = \left[w_{x_k}, w_{y_k}\right]^T = [\ddot{x}_k, \ddot{y}_k]^T \quad (4)$$

The mode transition matrix of the model is equal to F and the vector U is accurately measured with the navigation sensations by Equation (5).

$$F_k = \nabla_x f(x_k)|_{x_k=\hat{x}_{k-1}} = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, G_k = \begin{bmatrix} \frac{T^2}{2} & 0 \\ 0 & \frac{T^2}{2} \\ T & 0 \\ 0 & T \end{bmatrix} \quad (5)$$

$$U_{k-1,k} = \begin{bmatrix} u_{k1} \\ u_{k2} \\ u_{k3} \\ u_{k4} \end{bmatrix} = \begin{bmatrix} x_k^o - x_{k-1}^o - T\dot{x}_{k-1}^o \\ y_k^o - y_{k-1}^o - T\dot{y}_{k-1}^o \\ \dot{x}_k^o - \dot{x}_{k-1}^o \\ \dot{y}_k^o - \dot{y}_{k-1}^o \end{bmatrix} \quad (6)$$

Also, the nonlinear equations of the measurement vector with the mean of zero and the unknown initial values of the BOT problem are (7).

$$x_0 = r_0 \cdot \sin(z_0), r_0 = \sqrt{x_0^2 + y_0^2} \quad (7)$$
$$y_0 = r_0 \cdot \cos(z_0)$$

It is noteworthy that the nonlinear equations of measurement of the target side are considered as (8).

$$z_k = h(X_k) + v_k = \beta_k + v_k \quad (8)$$
$$h(X_k) = \tan^{-1}\left(\frac{x}{y}\right)$$
$$x = r \cdot \sin(z)$$
$$y = r \cdot \cos(z)$$
$$r = \sqrt{x^2 + y^2}$$

Exponential stability and interest analysis of $\mathbf{L_2}$ system

The aim is to find characteristics to prove the exponential stability and L_2 interest rate analysis of the system, which in this article deals with integral kernels. Consider system (9).

$$\dot{x}(t) = Ax(t) + A_d \int_0^\infty K(\theta)x(t-\theta-\tau)d\theta \quad (9)$$

Assume that the kernel satisfies (10) and is A or A_0 Hertz.

$$A_0 = A + A_d \int_0^\infty K(\theta)d\theta \quad (10)$$

To analyze the exponential stability of a high system with a convergence rate δ <δ_0, the Lyapunov function (11) is proposed.

$$V(t) = V_P(t) + V_G(t) + V_H(t), V_P(t) = x^T(t)Px(t) \quad (11)$$
$$V_G(t) = \int_0^\infty \int_{t-\theta-\tau}^{t} e^{-2\delta(t-s)}|K(\theta)|x^T(s)Gx(s)dsd\theta$$
$$V_H(t) = \int_0^\infty \int_0^{\theta+\tau} \int_{t-\lambda}^{t} e^{-2\delta(t-s)}|K(\theta)|\dot{x}^T(s)H\dot{x}(s)dsd\lambda d\theta$$

Where the matrices P, G and H are fixed and positive.
The sentence V_G (t) with $\delta \neq 0$ develops the classical result for the exponential stability mode with δ rate. Also, if A is Hertz, this sentence compensates for the effect of the delayed sentence in the above system. Similarly, the sentence V_H (t) extends the result to the exponential stability mode with δ rate, and if A_0 is Hertz, it compensates for the integral sentence in system (9). Of course, this sentence can also improve the results for a case where A is Hertz. Since the Lyapunov function is dependent on x., The primary function must be derivative.
In the following, conditions are extracted that satisfy the inequality (12) in order for the system to respond to the initial conditions (13).

$$\dot{V}(t) + 2\delta V(t) \leq 0 \quad (12)$$
$$\phi \in C^1(-\infty, 0] \quad (13)$$

In this case, the stability of the system is guaranteed. Therefore, the system response must have condition (14).

$$x^T(t)Px(t) \leq V(t) \quad (14)$$
$$\leq e^{-2\delta t}V(0), t \geq 0$$

Where for all $\delta \in (0, \delta_0)$ there is a relation (15).

$$V(0) \leq \lambda_{max}(P)|\phi(0)|^2 + \lambda_{max}(G)\int_0^\infty |K(\theta)|(\theta+\tau)d\theta \quad (15)$$
$$+\lambda_{max}(H)\int_0^\infty |K(\theta)|\frac{(\theta+\tau)^2}{2}d\theta \parallel \dot{\phi} \parallel_C$$

Derived from V in the direction of the relational system (16) is obtained.

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$$\dot{V}(t) + 2\delta V(t) = 2x^T(t)P\big[Ax(t) + A_d \int_0^\infty K(\theta)x(t-\theta-\tau \quad 2\delta x^T(t)Px(t) + \int_0^\infty |K(\theta)|d\theta x^T(t)Gx(t) - \int_0^\infty e^{-2\delta(\theta+\tau)}|K(\theta)|x^T(t-\theta-\tau)Gx(t-\theta-\tau)d\theta + \int_0^\infty (\theta+\tau)|K(\theta)|d\theta \dot{x}^T(t)H\dot{x}(t) - \int_0^\infty \int_{t-\theta-\tau}^t e^{-2\delta(\theta+\tau)}|K(\theta)|\dot{x}^T(s)H\dot{x}(s)dsd\theta \tag{16}$$

Assume that definitions (17) and (18) are considered.

$$K_{0\delta} = \int_0^\infty e^{2\delta(\theta+\tau)}|K(\theta)|d\theta, K_{00} = K_{0\delta}|_{\delta=0}, \tag{17}$$

$$K_{1\delta} = \int_0^\infty e^{2\delta(\theta+\tau)}|K(\theta)|(\theta+\tau)d\theta, K_{10} = K_{1\delta}|_{\delta=0} \tag{18}$$

Now, using the Jensen integral inequalities [20], equations (19) and (20) are obtained.

$$-\int^\infty e^{-2\delta(\theta+\tau)}|K(\theta)|x^T(t-\theta-\tau)Gx(t-\theta-\tau)d\theta \le -K_{0\delta}^{-1}\int_0^\infty K(\theta)x^T(t-\theta-\tau)d\theta G\int_0^\infty K(\theta)x(t-\theta-\tau)d \tag{19}$$

$$\int_{t-\theta-\tau}^t e^{-2\delta(\theta+\tau)}|K(\theta)|\dot{x}^T(s)H\dot{x}(s)dsd\theta \le {}^1\int_0^\infty \int_{t-\theta-\tau}^t K(\theta)\dot{x}^T(s)dsd\theta H\int_0^\infty \int_{t-\theta-\tau}^t K(\theta)\dot{x}(s)dsd\theta \tag{20}$$

With variable definition (21):

$$\eta(t) = \mathrm{col}\left\{x(t), \int_0^\infty K(\theta)x(t-\theta-\tau)d\theta\right\} \tag{21}$$

A relation (22) can be obtained.

$$\dot{V}(t) + 2\delta V(t) \le \eta^T(t)\begin{bmatrix}\Phi_{00} & PA_d + K_{18}^{-1}K_{00}H \\ * & -K_{08}^{-1}G - K_{18}^{-1}H\end{bmatrix}\eta(t) + K_{10}\eta^T(t)\begin{bmatrix}A^T \\ A_d^T\end{bmatrix}H\begin{bmatrix}A^T \\ A_d^T\end{bmatrix}^T\eta(t), \tag{22}$$

That

$$\Phi_{00} = PA + A^TP + 2\delta P + K_{00}G - K_{1\delta}^{-1}K_{00}^2H \tag{23}$$

LMI (24) is obtained by applying saline supplement [21]. As a result, the LMI obtained in Equation (24) ensures that Equation (25) is established.

$$\begin{bmatrix}\Phi_{00} & PA_d + K_{18}^{-1}K_{00}H & K_{10}A^TH \\ * & -K_{0\delta}^{-1}G - K_{18}^{-1}H & K_{10}A_d^TH \\ * & * & -K_{10}H\end{bmatrix} < 0 \tag{24}$$

$$\dot{V}(t) + 2\delta V \le 0 \tag{25}$$

Conclusion: Suppose that certain positive matrices $P, G, H \in \mathbb{R}^n$exist such that LMI (24) holds, in which case system (9) with initial conditions $\phi \in C^1(-\infty, 0]$ the exponential stability will be with the convergence rate δ.

A matrix kernel [22] can be considered for system (9). However, in this case the numerical solution of constants (26) and (27) is complex.

$$K_{0\delta} = \int_0^\infty e^{2\delta(\theta+\tau)}|K(\theta)|d\theta, K_{00} = K_{0\delta}|_{\delta=0}, \tag{26}$$

$$K_{1\delta} = \int_0^\infty e^{2\delta(\theta+\tau)}|K(\theta)|(\theta+\tau)d\theta, K_{10} = K_{1\delta}|_{\delta=0} \tag{27}$$

For this reason, in proving the exponential stability in this paper, a special but important case of matrix kernel is considered in which the matrix kernel is considered as the sum of scalar kernels. As a result, the system is considered as (28).

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^m A_{di}\int_0^\infty K_i(\theta)x(t-\theta-\tau)d\theta \tag{28}$$

If LMI (24) holds for δ = 0, then system (9) is asymptotic for $K \in L_1[0, \infty)$ if $K_{10} < \infty$ is the result of the delay effect distributed here with only a few integral sentences in the stability conditions. Reflected. Such conditions are highly conservative because they ignore the details of the delay distribution. For the kernel mode with gamma distribution [23] which is discussed below, the kernel derivative will also be considered to achieve better results.

A simpler Lyapunov function can be used when the matrix is A Heroitz. In this case, if the Lyapunov function (3) H = 0 is applied in V, LMI (29) is obtained.

$$\begin{bmatrix}PA + A^TP + 2\delta P + K_{00}G & PA_d \\ * & -K_{0\delta}^{-1}G\end{bmatrix} < 0 \tag{29}$$

As a result, for all system responses with the initial function $\phi \in C(-\infty, 0]$, LMI (29) ensures that relations (30) and (31) are established with H = 0.

$$x^T(t)Px(t) \le V(t) \le e^{-2\delta t}V(0), t \ge 0 \tag{30}$$

$$V(0) \le \lambda_{max}(P)|\phi(0)|^2 + \lambda_{max}(G)\int_0^\infty |K(\theta)|(\theta+\tau)d\theta + \lambda_{max}(H)\int_0^\infty |K(\theta)|\frac{(\theta+\tau)^2}{2}d\theta \parallel \dot{\phi} \parallel_C. \tag{31}$$

It can easily be seen that for δ = 0 LMI (29) is a sufficient condition independent of the delay for the system (32) for r≥0.

$$\dot{x}(t) = Ax(t) \pm K_{00}A_dx(t-r) \tag{32}$$

The following three items can also be guaranteed:

1) The matrices A and A ± K_00 A_d are Hertz (in other words, for K≥0 the matrix is A_0 Hertz).
2) The eigenvalues of the matrix (33) are inside a single circle.

$$A^{-1}K_{00}A_d = A^{-1}\int_0^\infty |K(s)|dsA_d \tag{33}$$

3) The Scaled Small Gain Theorem [24] (34) is established.

$$\|G^{0.5}(sI - A)^{-1}A_dG^{-0.5}\|_\infty < 1/K_{00} \tag{34}$$

For δ = 0, A_d = G = I and a matrix K, LMI (29) is equivalent to inequality (35).

$$\|(sI - A)^{-1}\|_\infty < 1/K_{00} \tag{35}$$

This inequality is extracted in the references for the limited delay mode. The result can be easily extended to systems with multiple latency and scalar kernels.

Theorem: Consider the system given in Equation (28). Suppose there exists a δ_0> 0 such that the relations (35) and (36) are established and A_0 is Hertz.

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$$\|(sI-A)^{-1}\|_{\infty} < 1/K_{00} \quad (35)$$

Given $\delta \in (0,\delta_0)$ ($\delta = 0$), assume that the definite positive matrices $P, G_i, H_i \in \mathbb{R}^{n\times n}$ are such that LMI (37) with details (38) Be established.

$$\begin{bmatrix} \Phi_{00} & \Phi_{01} & \dots & \Phi_{0i} & A^T\left(\sum_{i=1}^m K_{10}^i H_i\right) \\ * & \Phi_{11} & \dots & 0 & A_{d1}^T\left(\sum_{i=1}^m K_{10}^i H_i\right) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & \Phi_{mm} & A_{dm}^T\left(\sum_{i=1}^m K_{10}^i H_i\right) \\ * & * & * & * & -\sum_{i=1}^m K_{10}^i H_i \end{bmatrix} < 0 \quad (37)$$

$$K_{0\delta}^i = \int_0^{\infty} e^{2\delta(\theta+\tau)}|K_i(\theta)|d\theta, K_{00}^i = K_{0\delta}^i\big|_{\delta=0}, \quad (38)$$
$$K_{1\delta}^i = \int_0^{\infty} e^{2\delta(\theta+\tau)}|K_i(\theta)|(\theta+\tau)d\theta, K_{10}^i = K_{1\delta}^i\big|_{\delta=0},$$
$$\Phi_{00} = PA + A^TP + 2\delta P + \sum_{i=1}^m \left[K_{00}^i G_i - \left(K_{18}^i\right)^{-1}\left(K_{00}^i\right)\right.$$
$$\Phi_{0i} = PA_{di} + \left(K_{1\delta}^i\right)^{-1} K_{00}^i H_i, \Phi_{ii} = -\left(K_{0\delta}^i\right)^{-1} G_i - \left(K_{1\delta}^i\right.$$

L_2 and ISS interest rate analysis Disturbed systems are two other simple extensions of the Lyaponov-Krasovsky method, where the L_2 interest rate of the system will be analyzed. Consider the turbulent version of the previous system by Equation (39).

$$\dot{x}(t) = Ax(t) + A_d\int_0^{\infty} K(\theta)x(t-\theta-\tau)d\theta + Bw(t), \quad (39)$$
$$z(t) = Cx(t),$$

Where $w(t) \in \mathbb{R}^{n_w}$ are perturbation vectors, $z(t) \in \mathbb{R}^{n_z}$ are controlled outputs, and B and C are fixed matrices. The L_2 gain of a high system is said to be less than $\gamma > 0$ if there is a relation (40) for the initial zero conditions of the system.

$$J = \int_0^{\infty} [z^T(t)z(t) - \gamma^2 w^T(t)w(t)]dt < 0, \quad 0 \neq w \in L_2[0,\infty) \quad (40)$$

For an inequality $\delta > 0$ (40) guarantees that J <0 and is therefore an internal exponential stability system.
By standard calculations, we can see that W (t) ≤0 and therefore J <0 hold if LMI (41) holds.

$$\begin{bmatrix} \Phi_{00} + C^TC & PA_d + K_{18}^{-1}K_{00}H & PB & K_{10}A^TH \\ * & -K_{0\delta}^{-1}G - K_{1\delta}^{-1}H & 0 & K_{10}A_d^TH \\ * & * & -\gamma^2 I & K_{10}B^TH \\ * & * & * & -K_{10}H \end{bmatrix} < 0, \quad (41)$$
$$\Phi_{00} = PA + A^TP + 2\delta P + K_{00}G - K_{18}^{-1}K_{00}^2 H$$

Simulation

Consider the BOT system mentioned in the second part of the paper with the non-Hurwitz A matrix and the kernel matrix (42).

$$\dot{x}(t) = Ax(t) + \int_0^h K(\theta)x(t-\theta)d\theta \quad (42)$$
$$A = \begin{bmatrix} 0.2 & 0.01 \\ 0 & -2 \end{bmatrix}, K(\theta) = \begin{bmatrix} -1-0.3\theta & 0.1 \\ 0 & -0.1 \end{bmatrix}$$

$$A_0 = A + \sum_{i=1}^m A_{di}\int_0^{\infty} K_i(\theta)d\theta \quad (36)$$

System (42) with $m = 2,\ \tau = 0,\ K_1 = K_2 = 0\ for\ \theta > h$ can be written as (43) in different ways.

$$\dot{x}(t) = Ax(t) + \int_0^h K(\theta)x(t-\theta)d\theta \quad (43)$$
$$A = \begin{bmatrix} 0.2 & 0.01 \\ 0 & -2 \end{bmatrix}, K(\theta) = \begin{bmatrix} -1-0.3\theta & 0.1 \\ 0 & -0.1 \end{bmatrix}$$

Here, two forms (44) and (45) can be considered for the mentioned system.

$$A_{d1} = \begin{bmatrix} -1 & 0.1 \\ 0 & -0.1 \end{bmatrix}, A_{d2} = -\begin{bmatrix} 0.3 & 0 \\ 0 & 0 \end{bmatrix} \quad (44)$$
$$K_1 \equiv 1, K_2(\theta) = \theta, \theta \in [0,h]$$
$$A_{d1} = \begin{bmatrix} 0 & 0.1 \\ 0 & -0.1 \end{bmatrix}, A_{d2} = -\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad (45)$$
$$K_1 \equiv 1, K_2(\theta) = 1 + 0.3\theta, \theta \in [0,h]$$

Therefore, assuming equation (46) for the general system:

$$A_0 = A + \sum_{i=1}^2 A_{di}\int_0^{\infty} K_i(\theta)d\theta \quad (46)$$

In this system for h≥0.195 the matrix is A_0 Hurwitz. In the reference [25], using an analytical method, the delay interval for the asymptotic stability of the system h∈ [0.195,1.71] is obtained, while with the LMIs concluded in this paper with 15 scalar variables, the exponential stability of the system with the model (45) for h∈ [0.207,1.455] and with model (46) for h∈ [0.195,1.442]. Thus the system is exponentially stable for h∈ [0.195,1.455].
Assuming h = 1, it is observed that they can be solved by the existing LMIs with 15 decision variables maximum for the convergence rate $\delta_{max} = 0.433$ with model (44) and $\delta_{max} = 0.593$ with model (45). Therefore, the system is exponentially stable with a convergence rate of 0.593. Also note that the system in this example has a triangular structure. It is therefore stable if the two scalar systems (47) and (48) are stable.

$$\dot{x}_1(t) = 0.2x_1(t) - \int_{-h}^0 (1-0.3s)x_1(t+s)ds \quad (47)$$
$$\dot{x}_2(t) = -2x_2(t) - 0.1\int_{-h}^0 x_2(t+s)ds \quad (48)$$

Corresponding to models (47) and (48), the model for x_1 can be expressed as a system with two delays as (49).

$$A_{d1} = -1 = -K_1(\theta) \text{ and } A_{d2} = -0.3, K_2(\theta) = \theta \quad (49)$$

Or model with a system with only one delay as (50).

$$A_d = -1 \text{ and } K(\theta) = 1 + 0.3\theta(\theta \in [0,h]) \quad (50)$$

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The LMIs obtained in this paper with 5 and 3 scalar variables ensure the exponential stability of the system for h∈ [0.207,1.455] and h∈ [0.195,1.442], respectively.

Consider the BOT system mentioned in the second section with parameters (51).

$$A = 0.8, A_d = -41.8, B = 2, C = 1 \quad (51)$$
$$K(\theta) = 0 \text{ for } \theta > h$$
$$, K(\theta) = \frac{3+20\theta+700\theta^2}{2-20\theta+800\theta^2} \text{ for } \theta \in [0, h]$$

In this system for h> 0.011659 we have A_0 <0. For h = 0.1, the minimum value of L2 of the system in the reference [26] is calculated to be 0.76, while using the LMI obtained in this paper, the lowest value of L2 of the system is 0.3223.

Now if we consider the same system with infinite delay and assume that the scalar kernel $K \in L_1[0,\infty)$ is given with relation (52).

$$K(\theta) = \frac{3+20\theta+700\theta^2}{2-20\theta+800\theta^2} e^{-10\theta} \text{ for } \theta \in [0, \infty) \quad (52)$$

In this case, by solving the obtained LMI, the minimum value of L_2 of the system is equal to γ_{min} = 0.41.

Conclusion

In this paper, using the simulation method by Lyapunov function, it is shown that the proposed method of scalar kernel matrix for designing the optimal path of BOT problem can be calculated with a unique control function independent of the initial unknown parameters. Movable was evaluated by maneuver. Utilizing high accuracy of control methods of delayed systems with infinitely distributed delays in initial modeling with utilization of stabilization methods and L_2 gain analysis due to high stability to unknown parameters and increasing the solution speed to solve the complexity of the solution is the main feature of this method. Another advantage of designing a route with the desired pursuit and approach is at the beginning of the maneuver. In addition, high convergence, speed and low computational volume (compared to other methods) were shown in BOT application problems. Although convergence time can be calculated in error-free measurement mode, despite the significant measurement error, the minimum time in this study has been calculated from simulation. Future research in this area includes calculating the optimal time for the desired convergence commensurate with the sensitive measurement error to ensure the online convergence of the BOT problem. Also, the study of the performance of the proposed method for maneuverable targets and finding quick adaptive criteria appropriate to the functions of sustainability criteria for the convergence of maneuverable target estimators is considered in further research.

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